Split-Block Design

Fertility Trial Example

Petersen, 1994, p. 147ff.

Factors:

Block (B)	3	
Phosphate (P)	2	25, 50 kg/ha
Potash (K)	3	0, 25, 50 kg/ha

Split-Block Design

Fertility Trial Example – Barley Forage

Petersen, 1994, p. 147ff.

Field Plot Layout: Y = Yield (kg/plot)

	K_3	K_1	K_2
P_1	56	32	49
P_2	67	54	58

	K_1	K_3	K_2
P_2	38	62	50
P_1	52	72	64

	K_2	K_1	K_3
P_2	54	44	51
P_1	63	54	68

Fertility Trial

Expected Mean Squares

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + P_j + BP_{ij} + \omega_{(ij)} + K_k + BK_{ik} + \lambda_{(ik)} + PK_{jk} + BPK_{ijk}$$

Source	b R i	p F j	k F k	EMS
B_i	1	p	k	$\sigma^2 + p\sigma^2_\lambda + k\sigma^2_\omega + pk\sigma^2_\delta + pk\sigma^2_B$
$\delta_{(i)}$	1	p	k	$\sigma^2 + p\sigma^2_\lambda + k\sigma^2_\omega + pk\sigma^2_\delta$
P_j	b	0	k	$\sigma^2 + k\sigma^2_\lambda + k\sigma^2_{BP} + bk\Phi(P)$
BP_{ij}	1	0	k	$\sigma^2 + k\sigma^2_\lambda + k\sigma^2_{BP}$
$\omega_{(ij)}$	1	1	k	$\sigma^2 + k\sigma^2_\lambda$
K_k	b	p	0	$\sigma^2 + p\sigma^2_\lambda + p\sigma^2_{BK} + bp\Phi(K)$
BK_{ik}	1	p	0	$\sigma^2 + p\sigma^2_\lambda + p\sigma^2_{BK}$
$\lambda_{(ik)}$	1	p	1	$\sigma^2 + p\sigma^2_\lambda$
PK_{jk}	b	0	0	$\sigma^2 + \sigma^2_{BPK} + b\Phi(PK)$
BPK_{ijk}	1	0	0	$\sigma^2 + \sigma^2_{BPK}$

Fertility Trial

GLM Analysis

```
proc glm;
  class B P K;
  model yield = B P B*P K B*K P*K B*P*K;
  test h=P e=B*P;
  test h=K e=B*K;
  test h=P*K e=B*P*K;
run;
```

Fertility Trial
GLM Analysis

Dependent Variable: Yield

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	17	1833.777778	107.869281	.	.
Error	0	0.000000	.		
Corrected Total	17	1833.777778			

R-Square	Coeff Var	Root MSE	Yield Mean
1.000000	.	.	54.88889

Source	DF	Type III SS	Mean Square	F Value	Pr > F
B	2	45.7777778	22.8888889	.	.
P	1	56.8888889	56.8888889	.	.
B*P	2	693.7777778	346.8888889	.	.
K	2	885.7777778	442.8888889	.	.
B*K	4	78.2222222	19.5555556	.	.
P*K	2	19.1111111	9.5555556	.	.
B*P*K	4	54.2222222	13.5555556	.	.

Tests of Hypotheses Using the Type III MS for B*P as an Error Term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
P	1	56.88888889	56.88888889	0.16	0.7247

Tests of Hypotheses Using the Type III MS for B*K as an Error Term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
K	2	885.7777778	442.8888889	22.65	0.0066

Tests of Hypotheses Using the Type III MS for B*P*K as an Error Term

Source	DF	Type III SS	Mean Square	F Value	Pr > F
P*K	2	19.11111111	9.55555556	0.70	0.5467

Fertility Trial Example
Standard Errors

Main Effects Means

SED for P mean comparisons:

$$SED = \sqrt{\frac{2(346.889)}{3(3)}} = 8.78$$

SED for K mean comparisons:

$$SED = \sqrt{\frac{2(19.556)}{3(2)}} = 2.55$$

Fertility Trial Example

Standard Errors

Interaction Means

SED for P means at the same K treatment level:

$$SED = \sqrt{\frac{2[(3-1)13.556 + 346.889}{3(3)}} = 9.12$$

SED for K means at the same P treatment level:

$$SED = \sqrt{\frac{2[(2-1)13.556 + 19.556]}{3(2)}} = 3.32$$

SED for P means at different K treatment levels:

$$SED = \sqrt{\frac{2[(2(3)-2-3)(13.556 + 2(346.889) + 3(19.556)]}{3(2)(3)}} = 9.22$$

Fertility Trial Example

MIXED Analysis

```
proc mixed;
  class B P K;
  model yield = B P K P*K;
  random B*P B*K;
  lsmeans P K P*K / pdiff;
  contrast 'K linear' K -1 0 1;
  contrast 'K quad' K 1 -2 1;
run;
```

Note: B effect is included in model statement and is considered fixed. Interactions with B, however, are considered random. Treating B as random results in different SEs because σ^2_B becomes part of the expected variance.

Fertility Trial Example

MIXED Analysis

The Mixed Procedure

Differences of Least Squares Means

Effect	P	K	_P	_K	Estimate	Standard Error	DF	t Value	Pr > |t|
P	1		2		3.5556	8.7799	2	0.40	0.7247
K		1		2	-10.6667	2.5531	4	-4.18	0.0139
K		1		3	-17.0000	2.5531	4	-6.66	0.0026
K		2		3	-6.3333	2.5531	4	-2.48	0.0682
P*K	1	1	1	2	-12.6667	3.3222	4	-3.81	0.0189
P*K	1	1	1	3	-19.3333	3.3222	4	-5.82	0.0043
P*K	1	1	2	1	0.6667	9.1165	4	0.07	0.9452
P*K	1	1	2	2	-8.0000	9.2256	4	-0.87	0.4348
P*K	1	1	2	3	-14.0000	9.2256	4	-1.52	0.2037
P*K	1	2	1	3	-6.6667	3.3222	4	-2.01	0.1152
P*K	1	2	2	1	13.3333	9.2256	4	1.45	0.2219
P*K	1	2	2	2	4.6667	9.1165	4	0.51	0.6357
P*K	1	2	2	3	-1.3333	9.2256	4	-0.14	0.8921
P*K	1	3	2	1	20.0000	9.2256	4	2.17	0.0960
P*K	1	3	2	2	11.3333	9.2256	4	1.23	0.2866
P*K	1	3	2	3	5.3333	9.1165	4	0.59	0.5899
P*K	2	1	2	2	-8.6667	3.3222	4	-2.61	0.0595
P*K	2	1	2	3	-14.6667	3.3222	4	-4.41	0.0116
P*K	2	2	2	3	-6.0000	3.3222	4	-1.81	0.1452

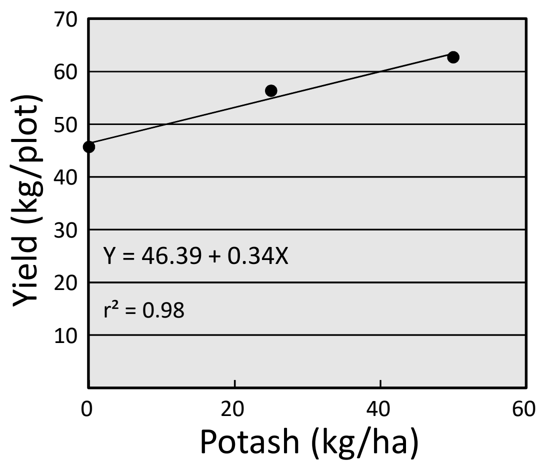
Fertility Trial Example

MIXED Analysis

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
K linear	1	4	44.34	0.0026
K quad	1	4	0.96	0.3826

Fertility Trial Example
MIXED Analysis



Split-Plot Design
2 Whole-Plot Factors

Small Grain Forage Example

Whole plots:

1) Grain - wheat, rye, triticale
2) Legume - hairy vetch, Austrian winter pea

Subplots:

Harvest date - boot, milk, hard dough

Split-Plot Design
2 Whole-Plot Factors

Layout:

Blocks		1						2					
Whole	Grain	1		2		3		1		2		3	
plots	Legume	1	2	1	2	1	2	1	2	1	2	1	2
Subplots	Stage	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3

Model:

$$Y_{ijkl} = \mu + B_i + \delta_{(i)} + G_j + BG_{ij} + L_k + BL_{ik} + GL_{jk} + BGL_{ijk} + \omega_{(ijk)} + S_l + BS_{il} + GS_{jl} + BGS_{ijl} + LS_{kl} + BLS_{ikl} + GLS_{jkl} + BGLS_{ijkl}$$

Expected Mean Squares

Whole Plots

Error A: BG_{ij}, BL_{ik}, BGL_{ijk}

Sub Plots

Error B: BS_{il}, BGS_{ijl}, BLS_{ikl}, $BGLS_{ijkl}$

Source	b R i	g F j	l F k	s F l	EMS
B_i	1	g	l	s	$s\sigma^2_\omega + gls\sigma^2_\delta + gls\sigma^2_B$
$\delta_{(i)}$	1	g	l	s	$s\sigma^2_\omega + gls\sigma^2_\delta$
$\mathbf{G_j}$	b	0	l	s	$s\sigma^2_\omega + ls\sigma^2_{BG} + bls\Phi(G)$
BG_{ij}	1	0	l	s	$s\sigma^2_\omega + ls\sigma^2_{BG}$
$\mathbf{L_k}$	b	g	0	s	$s\sigma^2_\omega + gs\sigma^2_{BL} + bgs\Phi(L)$
BL_{ik}	b	g	0	s	$s\sigma^2_w + gs\sigma^2_{BL}$
$\mathbf{GL_{jk}}$	b	0	0	s	$s\sigma^2_\omega + s\sigma^2_{BGL} + bs\Phi(GL)$
BGL_{ijk}	1	0	0	s	$s\sigma^2_\omega + s\sigma^2_{BGL}$
$\omega_{(ijk)}$	1	1	1	s	$s\sigma^2_\omega$
$\mathbf{S_l}$	b	g	l	0	$gl\sigma^2_{BS} + bgl\Phi(S)$
BS_{il}	1	g	l	0	$gl\sigma^2_{BS}$
$\mathbf{GS_{jl}}$	b	0	l	0	$l\sigma^2_{BGS} + bl\Phi(GS)$
BGS_{ijl}	1	0	l	0	$l\sigma^2_{BGS}$
$\mathbf{LS_{kl}}$	b	g	0	0	$g\sigma^2_{BLS} + bg\Phi(LS)$
BLS_{ikl}	1	g	0	0	$g\sigma^2_{BLS}$
$\mathbf{GLS_{jkl}}$	b	0	0	0	$\sigma^2_{BGLS} + b\Phi(GLS)$
$BGLS_{ijkl}$	1	0	0	0	σ^2_{BGLS}

Pooled Expected Mean Squares

	Source	b R i	g F j	l F k	s F l	EMS
	B_i	1	g	*l*	s	$s\sigma^2_\omega + gl s\sigma^2_\delta + gl s\sigma^2_B$
Whole Plots	$\delta_{(i)}$	1	g	*l*	s	$s\sigma^2_\omega + gl s\sigma^2_\delta$
	$\mathbf{G_j}$	b	0	*l*	s	$s\sigma^2_\omega + l s\sigma^2_{BG} + bl s\Phi(G)$
	$\mathbf{L_k}$	b	g	0	s	$s\sigma^2_\omega + gs\sigma^2_{BL} + bgs\Phi(L)$
	$\mathbf{GL_{jk}}$	b	0	0	s	$s\sigma^2_\omega + s\sigma^2_{BGL} + bs\Phi(GL)$
Error A	BGL_{ijk}	1	0	0	s	$s\sigma^2_\omega + s\sigma^2_{BGL}$
Sub Plots	$\omega_{(ijk)}$	1	1	1	s	$s\sigma^2_\omega$
	$\mathbf{S_l}$	b	g	*l*	0	$gl\sigma^2_{BS} + bgl\Phi(S)$
	$\mathbf{GS_{jl}}$	b	0	*l*	0	$l\sigma^2_{BGS} + bl\Phi(GS)$
	$\mathbf{LS_{kl}}$	b	g	0	0	$g\sigma^2_{BLS} + bg\Phi(LS)$
	$\mathbf{GLS_{jkl}}$	b	0	0	0	$\sigma^2_{BGLS} + b\Phi(GLS)$
Error B	$BGLS_{ijkl}$	1	0	0	0	σ^2_{BGLS}

Split-Split-Plot Design

Switchgrass Establishment Example

<u>Treatment factors</u>:

Blocks

Whole plots - tillage, vs. no till

Subplots - cultivar

Subsubplots - herbicide treatment

Split-Split-Plot Design

Switchgrass Establishment Example

Layout:

Blocks		1				2				3			
Whole Plots	Tillage	1		2		1		2		1		2	
Subplots	Cultivar	1	2	1	2	1	2	1	2	1	2	1	2
Subsubplots	Herbicide	1	1	1	1	1	1	1	1	1	1	1	1
		2	2	2	2	2	2	2	2	2	2	2	2
		3	3	3	3	3	3	3	3	3	3	3	3

Linear additive model:

$$Y_{ijkl} = \mu + B_i + \delta_{(i)} +$$
$$T_j + BT_{ij} + \omega_{(ij)} +$$
$$C_k + BC_{ik} + TC_{jk} + BTC_{ijk} + \lambda_{(ijk)} +$$
$$H_l + BH_{il} + TH_{jl} + BTH_{ijl} + CH_{kl} + BCH_{ikl} + TCH_{jkl} + BTCH_{ijkl}$$

Expected Mean Squares

	Source	b R i	t F j	c F k	h F l	EMS
	B_i	1	t	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + tch\sigma^2_\delta + tch\sigma^2_B$
Whole Plots	$\delta_{(i)}$	1	t	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + tch\sigma^2_\delta$
	T_j	b	0	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + ch\sigma^2_{BT} + bch\Phi(T)$
Error A	BT_{ij}	1	0	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + ch\sigma^2_{BT}$
Sub Plots	$\omega_{(ij)}$	1	1	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega$
	C_k	b	t	0	h	$h\sigma^2_\lambda + th\sigma^2_{BC} + bth\Phi(C)$
	BC_{ik}	b	t	0	h	$h\sigma^2_\lambda + th\sigma^2_{BC}$
	TC_{jk}	b	0	0	h	$h\sigma^2_\lambda + h\sigma^2_{BTC} + bh\Phi(TC)$
Error B	BTC_{ijk}	1	0	0	h	$h\sigma^2_\lambda + h\sigma^2_{BTC}$
Sub Sub Plots	$\lambda_{(ijk)}$	1	1	1	h	$h\sigma^2_\lambda$
	H_l	b	t	c	0	$tc\sigma^2_{BH} + btc\Phi(H)$
	BH_{il}	1	t	c	0	$tc\sigma^2_{BH}$
	TH_{jl}	b	0	c	0	$c\sigma^2_{BTH} + bc\Phi(TH)$
	BTH_{ijl}	1	0	c	0	$c\sigma^2_{BTH}$
	CH_{kl}	b	t	0	0	$t\sigma^2_{BCH} + bt\Phi(CH)$
	BCH_{ikl}	1	t	0	0	$t\sigma^2_{BCH}$
	TCH_{jkl}	b	0	0	0	$\sigma^2_{BTCH} + b\Phi(TCH)$
Error C	$BTCH_{ijkl}$	1	0	0	0	σ^2_{BTCH}

Pooled Expected Mean Squares

	Source	b R i	t F j	c F k	h F l	EMS
	B_i	1	t	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + tch\sigma^2_\delta + tch\sigma^2_B$
Whole Plots	$\delta_{(i)}$	1	t	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + tch\sigma^2_\delta$
	T_j	b	0	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + ch\sigma^2_{BT} + bch\Phi(T)$
Error A	BT_{ij}	1	0	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega + ch\sigma^2_{BT}$
Sub Plots	$\omega_{(ij)}$	1	1	c	h	$h\sigma^2_\lambda + ch\sigma^2_\omega$
	C_k	b	t	0	h	$h\sigma^2_\lambda + th\sigma^2_{BC} + bth\Phi(C)$
	TC_{jk}	b	0	0	h	$h\sigma^2_\lambda + h\sigma^2_{BTC} + bh\Phi(TC)$
Error B	BTC_{ijk}	1	0	0	h	$h\sigma^2_\lambda + h\sigma^2_{BTC}$
Sub Sub Plots	$\lambda_{(ijk)}$	1	1	1	h	$h\sigma^2_\lambda$
	H_l	b	t	c	0	$tc\sigma^2_{BH} + btc\Phi(H)$
	TH_{jl}	b	0	c	0	$c\sigma^2_{BTH} + bc\Phi(TH)$
	CH_{kl}	b	t	0	0	$t\sigma^2_{BCH} + bt\Phi(CH)$
	TCH_{jkl}	b	0	0	0	$\sigma^2_{BTCH} + b\Phi(TCH)$
Error C	$BTCH_{ijkl}$	1	0	0	0	σ^2_{BTCH}

Split-Plot In Time

Treatments:

Blocks (B)
Whole plots (W)
Split plots (S)

Split is conceptual, as treatment is applied to entire plot (or experimental unit) which is sampled in time (repeated measure).

Linear additive model:

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + W_j + BW_{ij} + \omega_{(ij)} + S_k + BS_{ik} + WS_{jk} + BWS_{ijk}$$

Split-Plot in Time

Grass-Legume Mixture Example

<u>Treatments</u>:

Blocks (B) 4 reps

Treatment - (T) 4 grass-legume mixtures

1) no legume control
2) birdsfoot trefoil
3) alfalfa
4) kura clover

Date - Repeated Measure -(D) 4 harvest dates

1) spring
2) early summer
3) mid summer
4) late summer

Split-Plot in Time

Grass-Legume Example

- Field layout is the same as a RCBD
- Entire plot is harvested at each interval and allowed to regrow before next harvest
- Need to assume that measurements collected from a plot at one harvest are uncorrelated with measurements collected from same plot at other harvests

Block	Plot 1	2	3	4
1	1	3	2	4
2	3	4	1	2
3	3	2	4	1
4	1	3	4	2

Grass-Legume Example

Expected Mean Squares

	Source	b R i	t F j	d F k	EMS
	B_i	1	t	d	$d\sigma^2_{\omega} + td\sigma^2_{\delta} + td\sigma^2_{B}$
	$\delta_{(i)}$	1	t	d	$d\sigma^2_{\omega} + td\sigma^2_{\delta}$
	T_j	b	0	d	$d\sigma^2_{\omega} + d\sigma^2_{BT} + bd\Phi(T)$
Error a	BT_{ij}	1	0	d	$d\sigma^2_{\omega} + d\sigma^2_{BT}$
	$\omega_{(ij)}$	1	1	d	$d\sigma^2_{\omega}$
	D_k	b	t	0	$t\sigma^2_{BD} + bt\Phi(D)$
Pool	BD_{ik}	1	t	0	$t\sigma^2_{BD}$
	WD_{jk}	b	0	0	$\sigma^2_{BTD} + b\Phi(TD)$
Error b	BTD_{ijk}	1	0	0	σ^2_{BTD}

Split-Plot in Time

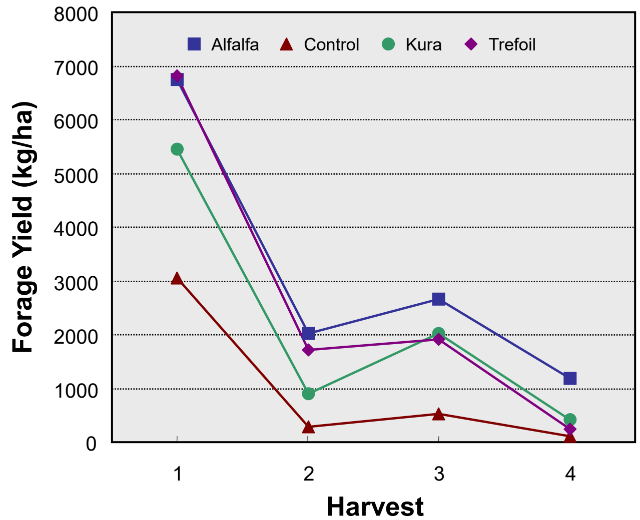
Grass-Legume Mixture Example

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Blk	3	505996.5	168665.5	1.22	
Trt	3	40986209	13662070	253.96	<.0001
Error a	9	484157.4	53795.3	0.39	
Date	3	2.41E+08	80241422	578.19	<.0001
Trt*Date	9	15762104	1751345	12.62	<.0001
Error b	36	4996075	138779.9		

This analysis is OK as long as our assumptions hold. What happens if repeated measurements are correlated?

Split-Plot in Time

Grass-Legume Mixture Example



Split-Plot in Time

Grass-Legume Mixture Example

trt*date Effect Sliced by date for yield

Date	DF	SS	Mean Square	F Value	Pr > F
1	3	36884379	12294793	88.59	<.0001
2	3	7402380	2467460	17.78	<.0001
3	3	9688544	3229515	23.27	<.0001
4	3	2773011	924337	6.66	.00011

$$SED = \sqrt{\frac{2[(4-1)(138779.9) + 53795.3]}{4(4)}} = 242.42$$

$$LSD = 2.028(242.42) = 491.63$$

Grass-Legume Mixture Example

Differences of Least Squares Means

Effect	trt	date	_trt	_date	Estimate	Error	DF	t Value	Pr > |t|
trt*date	Alfalfa	1	Control	1	3677.25	242.42	36	15.17	<.0001
trt*date	Alfalfa	1	Kura	1	1288	242.42	36	5.31	<.0001
trt*date	Alfalfa	1	Trefoil	1	-80	242.42	36	-0.33	0.7433
trt*date	Control	1	Kura	1	-2389.25	242.42	36	-9.86	<.0001
trt*date	Control	1	Trefoil	1	-3757.25	242.42	36	-15.5	<.0001
trt*date	Kura	1	Trefoil	1	-1368	242.42	36	-5.64	<.0001
trt*date	Alfalfa	2	Control	2	1723.75	242.42	36	7.11	<.0001
trt*date	Alfalfa	2	Kura	2	1123	242.42	36	4.63	<.0001
trt*date	Alfalfa	2	Trefoil	2	296.25	242.42	36	1.22	0.2296
trt*date	Control	2	Kura	2	-600.75	242.42	36	-2.48	0.018
trt*date	Control	2	Trefoil	2	-1427.5	242.42	36	-5.89	<.0001
trt*date	Kura	2	Trefoil	2	-826.75	242.42	36	-3.41	0.0016
trt*date	Alfalfa	3	Control	3	2135.5	242.42	36	8.81	<.0001
trt*date	Alfalfa	3	Kura	3	641.25	242.42	36	2.65	0.012
trt*date	Alfalfa	3	Trefoil	3	759.5	242.42	36	3.13	0.0034
trt*date	Control	3	Kura	3	-1494.25	242.42	36	-6.16	<.0001
trt*date	Control	3	Trefoil	3	-1376	242.42	36	-5.68	<.0001
trt*date	Kura	3	Trefoil	3	118.25	242.42	36	0.49	0.6287
trt*date	Alfalfa	4	Control	4	1077	242.42	36	4.44	<.0001
trt*date	Alfalfa	4	Kura	4	767.5	242.42	36	3.17	0.0031
trt*date	Alfalfa	4	Trefoil	4	938	242.42	36	3.87	0.0004
trt*date	Control	4	Kura	4	-309.5	242.42	36	-1.28	0.2099
trt*date	Control	4	Trefoil	4	-139	242.42	36	-0.57	0.5699
trt*date	Kura	4	Trefoil	4	170.5	242.42	36	0.7	0.4864